

**337351(14)**

**B. E. (Third Semester) Examination, April-May 2020/**

**(New Scheme)**

*NOV-DEC 2020*

**(Mech., Production and Automobile Engg. Branch)**

**MATHEMATICS-III**

***Time Allowed : Three hours***

***Maximum Marks : 80***

***Minimum Pass Marks : 28***

***Note : Attempt all questions. Part (a) of each unit is compulsory. Attempt any two parts from (b), (c) and (d) of each unit.***

**Unit-I**

1. (a) If  $f(x) = x$  is defined in the interval  $[0, 2\pi]$ , find

the value of  $a_0$ :

2

(b) Prove that  $x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}$ , in the

interval  $-\pi < x < \pi$ . 7

(c) If  $f(x) = |\cos x|$ , expand  $f(x)$  as a Fourier series

in the interval  $(-\pi, \pi)$ . 7

(d) Obtain the 1st three coefficients in the Fourier cosine

series for  $y$ , where  $y$  is given in the following table. 7

$x$	:	0	1	2	3	4	5
$y$	:	4	8	15	7	6	2

**Unit-II**

2. (a) Find the Laplace transform of  $e^{-3t} \sin 5t \cdot \sin 3t$ . 2

(b) Prove that  $\int_0^{\infty} \frac{e^{-t} \sin^2 t}{t} dt = \frac{1}{4} \log 5$  7

(c) Using convolution theorem, evaluate : 7

$$L^{-1} \left\{ \frac{1}{s^2 (s^2 + a^2)} \right\}$$

(d) Use transform method to solve : 7

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} - 3y = \sin t$$

$$y = \frac{dy}{dt} = 0, \text{ when } t = 0$$

**Unit-III**

3. (a) Form the partial differential equation : 2

$$z = f(x^2 - y^2)$$

(b) Solve the partial differential equation 7

$$x^2 (y-z)p + y^2 (z-x)q = z^2 (x-y)$$

(c) Solve : 7

$$(D^2 - DD' - 2D'^2)z = (y+1)e^x$$

(d) Solve the equation by the method of separation of variable : 7

$$3 \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0, \quad u(x, 0) = 4e^{-x}$$

**Unit-IV**

4. (a) Write the polar form of Cauchy Riemann equation. 2

(b) Determine the analytic function whose real part is

$$\frac{y''}{x^2 + y^2} \quad 7$$

(c) Find the residue of  $f(z) = \frac{z^3}{(z-1)^2(z-2)(z-3)}$  at

its poles and hence evaluate  $\oint_C f(z) dz$ , where

$$C : |z| = 2.5 \quad 7$$

(d) By integrating around a unit circle, evaluate 7

$$\int_0^{2\pi} \frac{d\theta}{17 - 8\cos\theta}$$

**Unit-V**

5. (a) Define Distribution function of the continuous random variable. 2

(b)  $x$  is a continuous random variable with probability density function given by :

$$\begin{aligned} f(x) &= Kx & (0 \leq x < 2) \\ &= 2K & (2 \leq x < 4) \\ &= -Kx + 6K & (4 \leq x < 6) \end{aligned}$$

Find  $K$  and mean value of  $x$ . 7

(c) Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or six? 7

(d) Fit Poisson's distribution to the following and calculate theoretical frequencies ( $e^{-0.5} = 0.61$ ). 7

Deaths	:	0	1	2	3	4
Frequency	:	122	60	15	2	1